Ableitung von f(x) = ln(x)

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(x)\right) &= \lim_{h\to 0}\frac{\ln(x+h)-\ln(x)}{h}\\
&= \lim_{h\to 0}\left(\frac{1}{h}\cdot\ln\left(\frac{x+h}{x}\right)\right)\\
&= \lim_{h\to 0}\left(\frac{1}{h}\cdot\ln\left(1+\frac{h}{x}\right)\right)\\
&= \lim_{h\to 0}\left(\frac{x}{x}\cdot\frac{1}{h}\cdot\ln\left(1+\frac{h}{x}\right)\right)\\
&= \lim_{h\to 0}\left(\frac{1}{x}\cdot\frac{x}{h}\cdot\ln\left(1+\frac{h}{x}\right)\right)\\
&= \lim_{h\to 0}\left(\frac{1}{x}\cdot\ln\left(1+\frac{h}{x}\right)^{\frac{x}{h}}\right)\\
&= \frac{1}{x}\cdot\lim_{h\to 0}\left(\ln\left(1+\frac{h}{x}\right)^{\frac{x}{h}}\right)\\
&= \frac{1}{x}\cdot\ln\overbrace{\left(\lim_{h\to 0}\left(1+\frac{h}{x}\right)^{\frac{x}{h}}\right)}^{\text{Definition von e}}\\
&= \frac{1}{x}\cdot\overbrace{\ln(e)}^{1}\\
&= \frac{1}{x}\qquad\blacksquare
\end{aligned}
$$